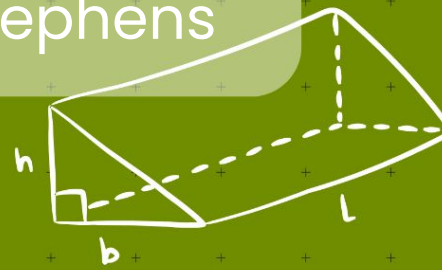
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RETHINKING MATH INTERVENTIONS

Fluency Beyond Memorization

Presenters: Nicole Bisgaard and Jenna Stephens



COMPASS
PARTNERS IN LEARNING

OUR TEAM

Jenna Stephens
jstephens@bhssc.org

Nicole Bisgaard
nbisgaard@bhssc.org

OUTCOMES

WHAT IS FLUENCY?

WHAT'S NOT WORKING

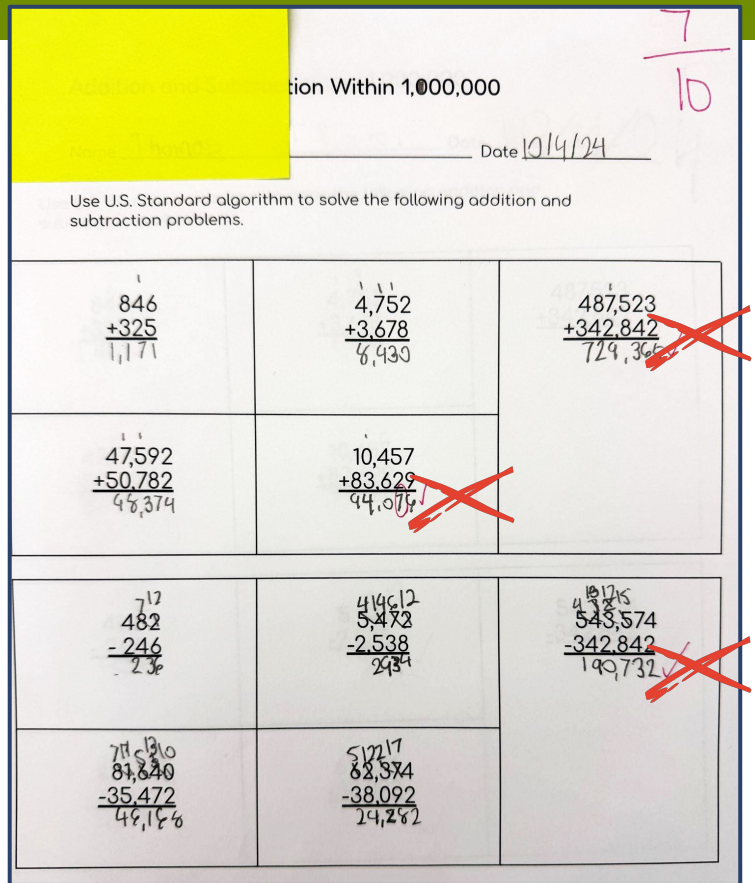
WHAT ACTUALLY BUILDS FLUENCY

36% of **4th graders** proficient at grade-level math.

26% of **8th graders** proficient at grade-level math.

NAEP 2022

Do these students need intervention?



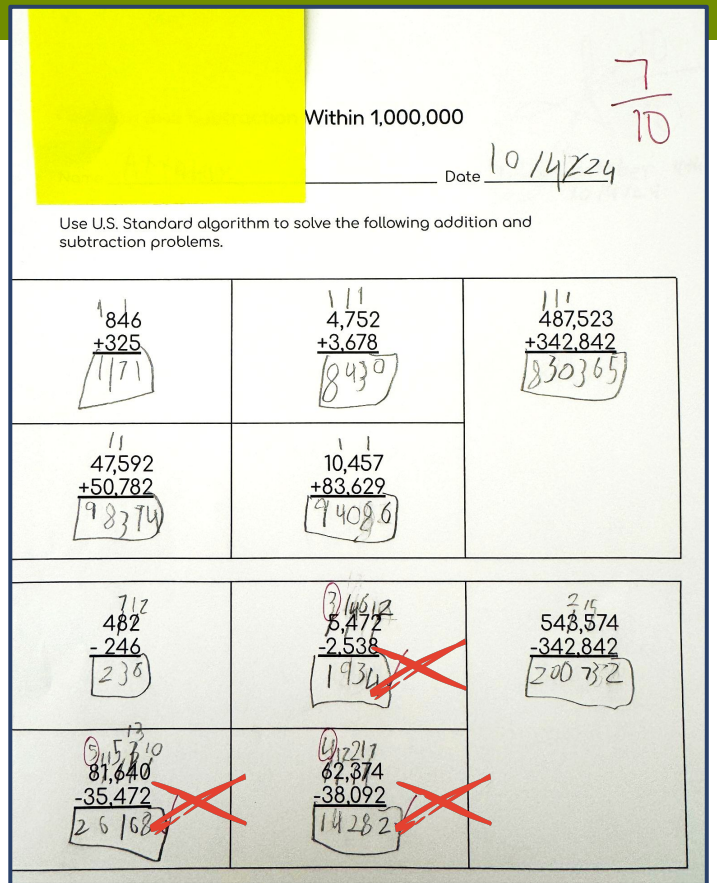
ion Within 1,000,000

Date 10/4/24

7/10

Use U.S. Standard algorithm to solve the following addition and subtraction problems.

846 +325 1,171	4,752 +3,678 8,430	487,523 +342,842 729,365
47,592 +50,782 98,374	10,457 +83,629 94,086	
482 - 246 236	5,472 -2,538 2934	543,574 -342,842 190,732
81,640 -35,472 46,168	82,374 -38,092 24,282	



Within 1,000,000

Date 10/4/24

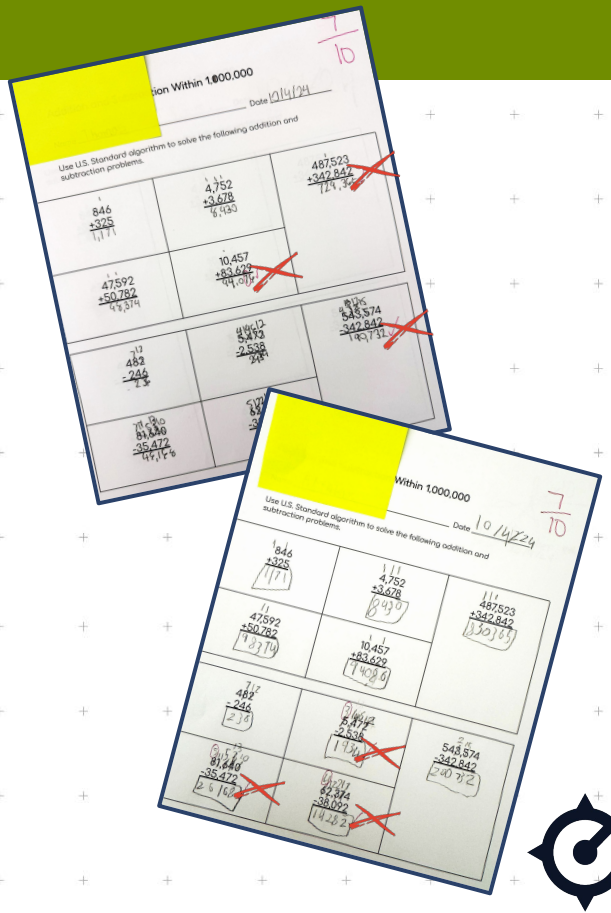
7/10

Use U.S. Standard algorithm to solve the following addition and subtraction problems.

846 +325 1171	4,752 +3,678 8430	487,523 +342,842 830365
47,592 +50,782 98374	10,457 +83,629 94086	
482 - 246 236	5,472 -2,538 1934	543,574 -342,842 200732
81,640 -35,472 26168	82,374 -38,092 14282	

Questions to ponder…

- ❑ Was this a good assessment tool?
- ❑ Was the assessment interpreted accurately?
- ❑ What is our definition of proficiency?
- ❑ Do we know what each student needs?



WHAT IS FLUENCY?

Notebook or Neighbor

How do you define *fluency*?

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What is the difference?

Find the product.

$\frac{1}{2} \times \frac{2}{3}$

Student A Thought Process:

"Yuck, I hate fractions."

Student B Thought Process:

"I wonder if I need a common denominator?"

What is the difference?

Find the product.

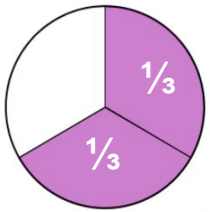
$\frac{1}{2} \times \frac{2}{3}$

Student C Thought Process:

"I know I need to multiply the numerators together and the denominators together. So 1x2=2 and 2x3=6 so the answer would be 2/6."

Student D Thought Process:

"I need to find what half of two-thirds is. If I have 2 "one-thirds" half of that would be one third (⅓)."



Fluency in math is...

"Being **flexible** with numbers, understanding **why** things work, and being able to **think** through problems to get correct answers."

-Christina Tondevold

What is fluency?

Fluency is...

being able to apply procedures

Efficiently **Flexibly** **Accurately**

NCTM 2001. SD State Standards

COMPASS

Mathematics | Grade 2

Grade 2 Overview

Operations and Algebraic Thinking

A. Represent subtractio
B. Add and su
C. Work with for multipl

Number and Opera

A. Understand
B. Use place v operations

Measurement and

A. Measure ar
B. Relate addition and subtraction to length.
C. Work with time a
D. Represent and int

Geometry

A. Reason with shap

Important Definitions an

- Fluency-skill in ca
- Know from memo

K-2 Common Addition and Subtraction Situations – Addition and Subtraction Problem Types Chart (see page 9)

Fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Know from Memory: quick, effortless, recall of facts

Strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

South Dakota K-5 Math Fluency Progression

Foundations	Conceptual understanding	**Fluently**	Skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
Developing	Conceptual understanding with strategies to operate	**Memorization**	Quick, effortless recall of facts.

Strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Four operations with **whole numbers.**

Kindergarten

Addition

- Within 20: Fluentlly add and subtract **within 5.**
- Within 20: Add and subtract **within 10** using objects and drawings.

Subtraction

- Within 20: Fluentlly add and subtract **within 5.**
- Within 20: Add and subtract **within 10** using objects and drawings.

1st Grade

Addition

- Within 20: Fluentlly add and subtract **within 10.**
- Within 20: Add and subtract **within 20.**
- Within 100: Add **within 100.**

Subtraction

- Within 20: Fluentlly add and subtract **within 10.**
- Within 20: Add and subtract **within 20.**
- Within 100: Subtract **within 100.** (multiples of 10)

2nd Grade

Addition

- Within 20: Fluently add/subtract using mental strategies **within 20.**
- Within 20: By end of year know all **sums within 20** from **memory.**
- Within 100: Fluently add/subtract **within 100.**
- Within 1000: Add/subtract **within 1000.**

Subtraction

- Within 20: Fluently add/subtract using mental strategies **within 20.**
- Within 20: By end of year know all **sums within 20** from **memory.**
- Within 100: Fluently add/subtract **within 100.**
- Within 1000: Add/subtract **within 1000.**

Multiplication

- Within 20: Foundations only: equal groups (objects to 20) and arrays (up to 5x5)

3rd Grade

Addition

- Within 1000: Fluently add/subtract **within 1000. (Algorithm included)**
- 1000+: Fluently add/subtract **within 1000. (Algorithm included)**

Subtraction

- Within 1000: Fluently add/subtract **within 1000. (Algorithm included)**
- 1000+: Fluently add/subtract **within 1000. (Algorithm included)**

Multiplication

- Within 100: Fluently multiply/divide **within 100.**
- Within 1000: Multiply **4x1-digit** and **2x2-digit.**

Division

- Within 100: Fluently multiply/divide **within 100.**
- Within 1000: Divide **4 by 1-digit.** (Whole digit remainders)

4th Grade

Addition

- Within 1000: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**
- 1000+: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**

Subtraction

- Within 1000: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**
- 1000+: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**

Multiplication

- Within 100: Know from **memory** all products of **2 one digit numbers.**
- Within 1000: Multiply **4x1-digit** and **2x2-digit.**

Division

- Within 100: Divide **4 by 1-digit.** (Whole digit remainders)
- Within 1000: Divide **4 by 1-digit.** (Whole digit remainders)

5th Grade

Addition

- Within 1000: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**
- 1000+: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**

Subtraction

- Within 1000: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**
- 1000+: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**

Multiplication

- Within 1000: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**
- 1000+: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**

Division

- Within 1000: Divide **4 by 2-digit numbers.**
- 1000+: Divide **4 by 2-digit numbers.**

6th Grade

Addition

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Subtraction

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Multiplication

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Division

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Four operations with **decimals.**

Kindergarten

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

1st Grade

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

2nd Grade

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

Multiplication

- Decimals: Exposure to decimals in terms of money

3rd Grade

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

Multiplication

- Decimals: Exposure to decimals in terms of money

Division

- Decimals: Exposure to decimals in terms of money

4th Grade

Addition

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

Subtraction

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

Multiplication

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

Division

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

5th Grade

Addition

- Decimals: Add/Subtract decimals to the hundredth.

Subtraction

- Decimals: Add/Subtract decimals to the hundredth.

Multiplication

- Decimals: Multiply/Divide decimals to the hundredth.

Division

- Decimals: Multiply/Divide decimals to the hundredth.

6th Grade

Addition

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Subtraction

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Multiplication

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Division

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Four operations with **fractions.**

Kindergarten

Addition

- Represent(visually & verbally) **halves & fourths.**

Subtraction

- Represent(visually & verbally) **halves & fourths.**

1st Grade

Addition

- Represent(visually & verbally) **halves, thirds, & fourths.** Whole Numbers

Subtraction

- Represent(visually & verbally) **halves, thirds, & fourths.** Whole Numbers

2nd Grade

Addition

- Represent(visually, verbally, numerically) **halves, thirds, & fourths, sixths, eighths.** Whole Numbers, comparisons, equivalent, benchmarks

Subtraction

- Represent(visually, verbally, numerically) **halves, thirds, & fourths, sixths, eighths.** Whole Numbers, comparisons, equivalent, benchmarks

Multiplication

- Represent(visually, verbally, numerically) **halves, thirds, & fourths, sixths, eighths.** Whole Numbers, comparisons, equivalent, benchmarks

3rd Grade

Addition

- Add/Subtract fractions with **like denominators** (including mixed numbers).
- Add fractions with denominators 10 and 100.

Subtraction

- Add/Subtract fractions with **like denominators** (including mixed numbers).
- Add fractions with denominators 10 and 100.

Multiplication

- Multiply whole number by fraction.

4th Grade

Addition

- Add/Subtract fractions with **unlike denominators** (including mixed numbers).

Subtraction

- Add/Subtract fractions with **unlike denominators** (including mixed numbers).

Multiplication

- Multiply (whole number x fraction) & (fraction x fraction) (including mixed numbers)

Division

- Divide (whole number by unit fraction) & (unit fraction by whole number)

5th Grade

Addition

- Multiply/Divide fractions by fractions. (including mixed numbers)

Subtraction

- Multiply/Divide fractions by fractions. (including mixed numbers)

Multiplication

- Multiply/Divide fractions by fractions. (including mixed numbers)

Division

- Multiply/Divide fractions by fractions. (including mixed numbers)

6th Grade

Addition

- Multiply/Divide fractions by fractions. (including mixed numbers)

Subtraction

- Multiply/Divide fractions by fractions. (including mixed numbers)

Multiplication

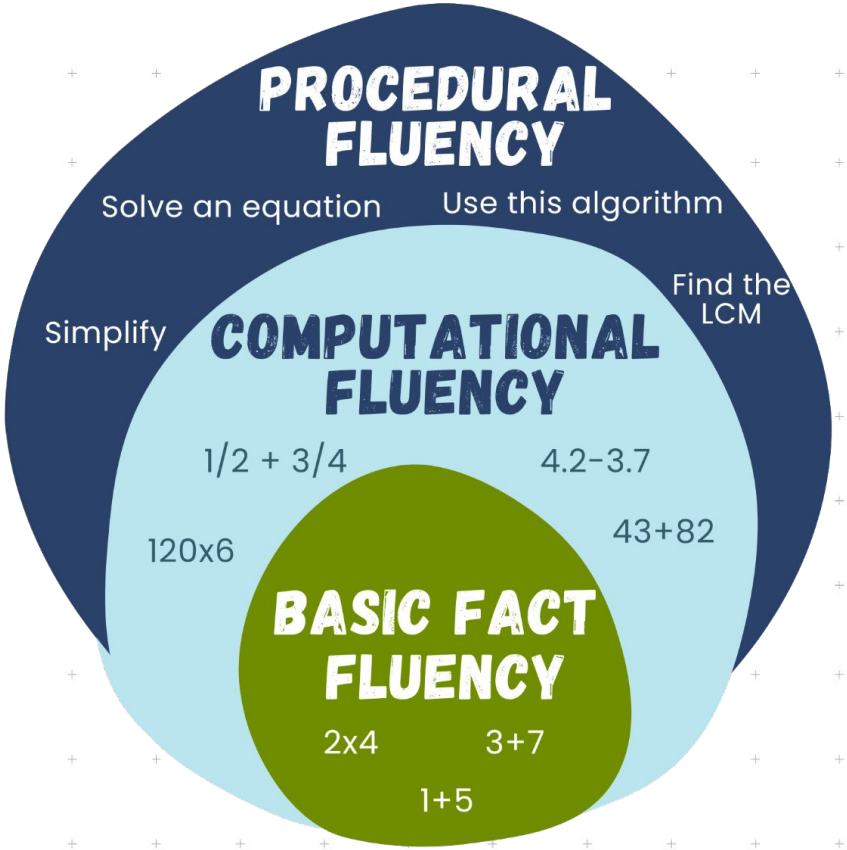
- Multiply/Divide fractions by fractions. (including mixed numbers)

Division

- Multiply/Divide fractions by fractions. (including mixed numbers)

3 Types of Fluency

When we think of fluency, we often just think of basic math facts. It is so much more than that... we **all** have a role in math fluency K-12!

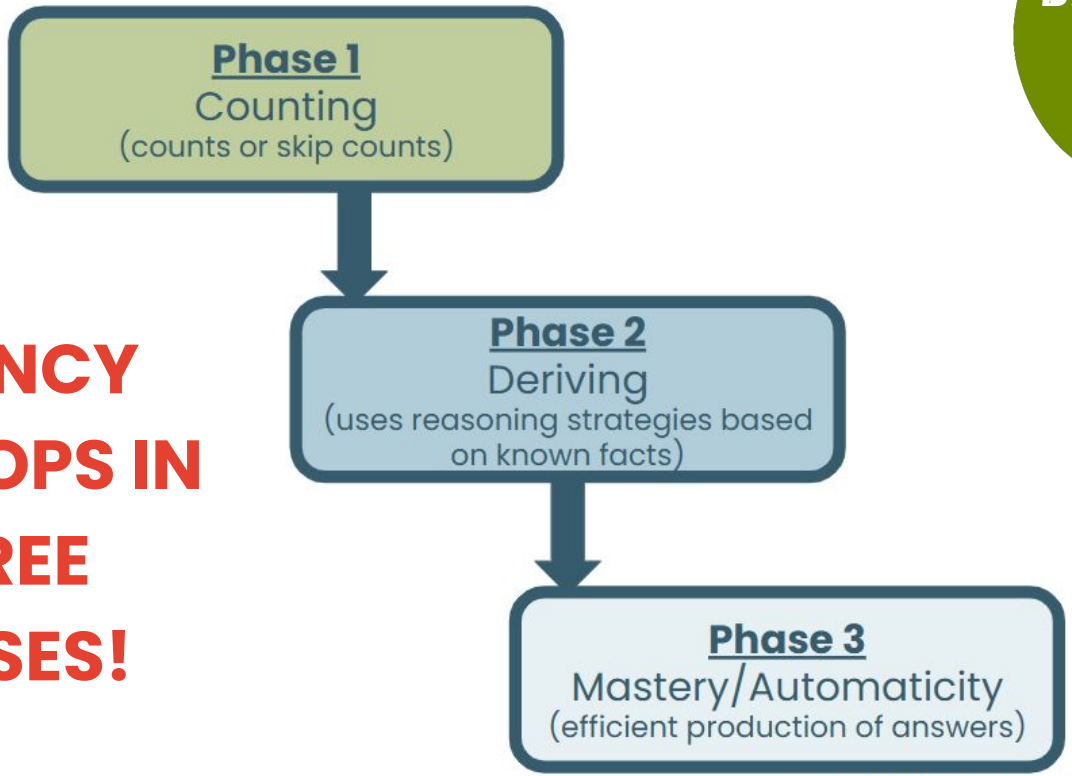


Basic Fact Fluency

BASIC FACT FLUENCY

2x4 3+7 1+5

FLUENCY DEVELOPS IN THREE PHASES!



@JBayWilliams, Baroody 2006

PROCEDURAL FLUENCY: 4 DECLARATIONS

1. Basic facts should be taught using **number relationships** and **reasoning strategies**, **not memorization**.
2. Procedural fluency requires having a **repertoire of strategies**.

Basic Fact Strategies

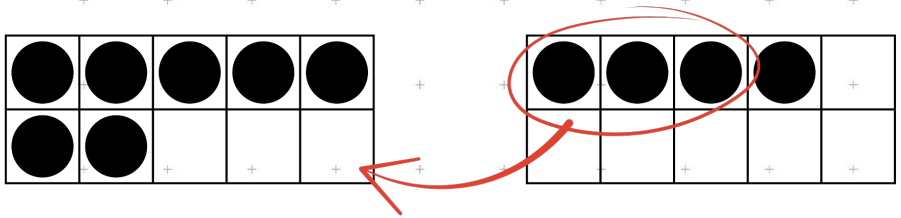
STRATEGY	BASIC FACT
Making 10	7+9=6+10=16
Compensation(Pretend-a-10)	7+9=7+10=17-1=16
Near Doubles	6+5=5+5+1=11
Think Addition	11-7→7+?=11
Doubling	4x7=2x7x2
Add-a-Group	6x7=(5x7)+7
Subtract-a-Group	9x8=10x8-8
Think Multiplication	45÷9→9x?=45
Break Apart	7x6=(2x6)+(5x6)=12+30=42

Basic Fact Fluency

7+4=_

Phase 1:
(counting)

Phase 2:
(deriving)



7+4=**7+3**+1=
10+1=11

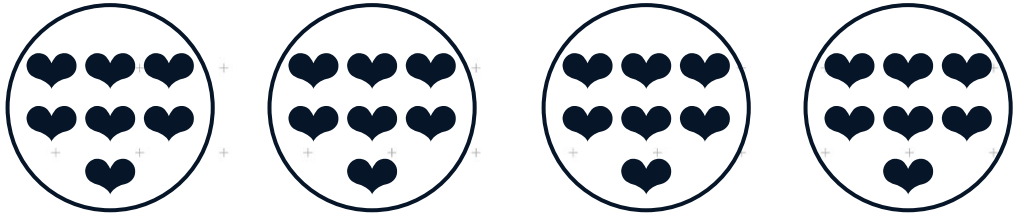
Phase 3:
(automaticity)

7+4 is **11**

Basic Fact Fluency

$4 \times 7 = \_$

Phase 1:
(counting)



Phase 2:
(deriving)

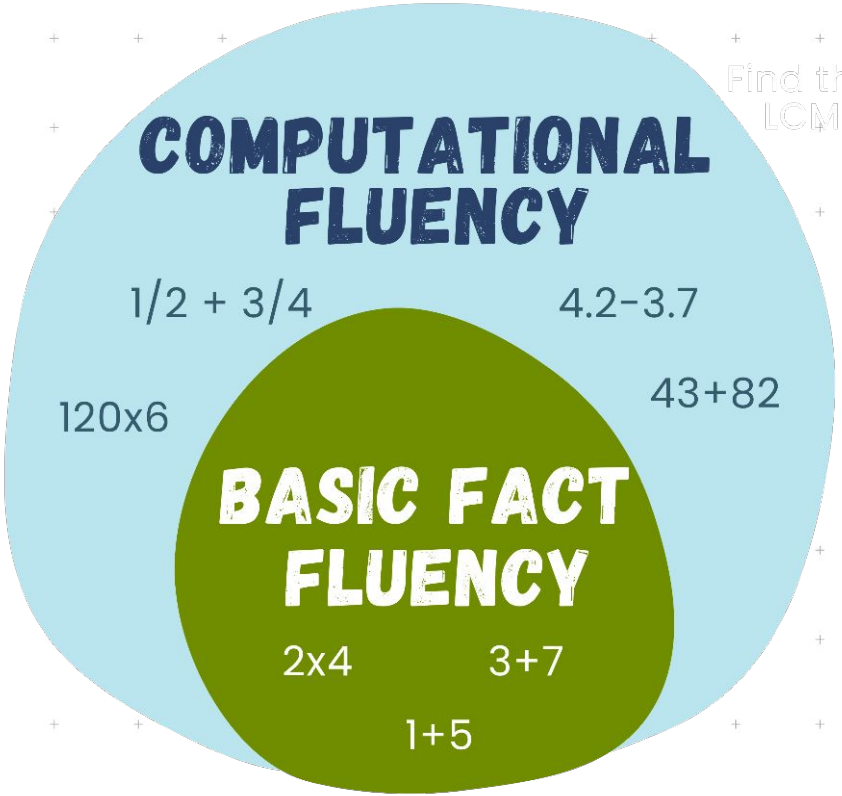
$4x7 = 2x7 + 2x7 =$
$14 + 14 = 28$

$4x7 = 4x5 + 4x2 =$
$20 + 8 = 28$

Phase 3:
(automaticity)

4x7 is **28**

Computational Fluency



$$9 + 7 = 10 + 6 = 16$$

$$99 + 7 = 100 + 6 = 106$$

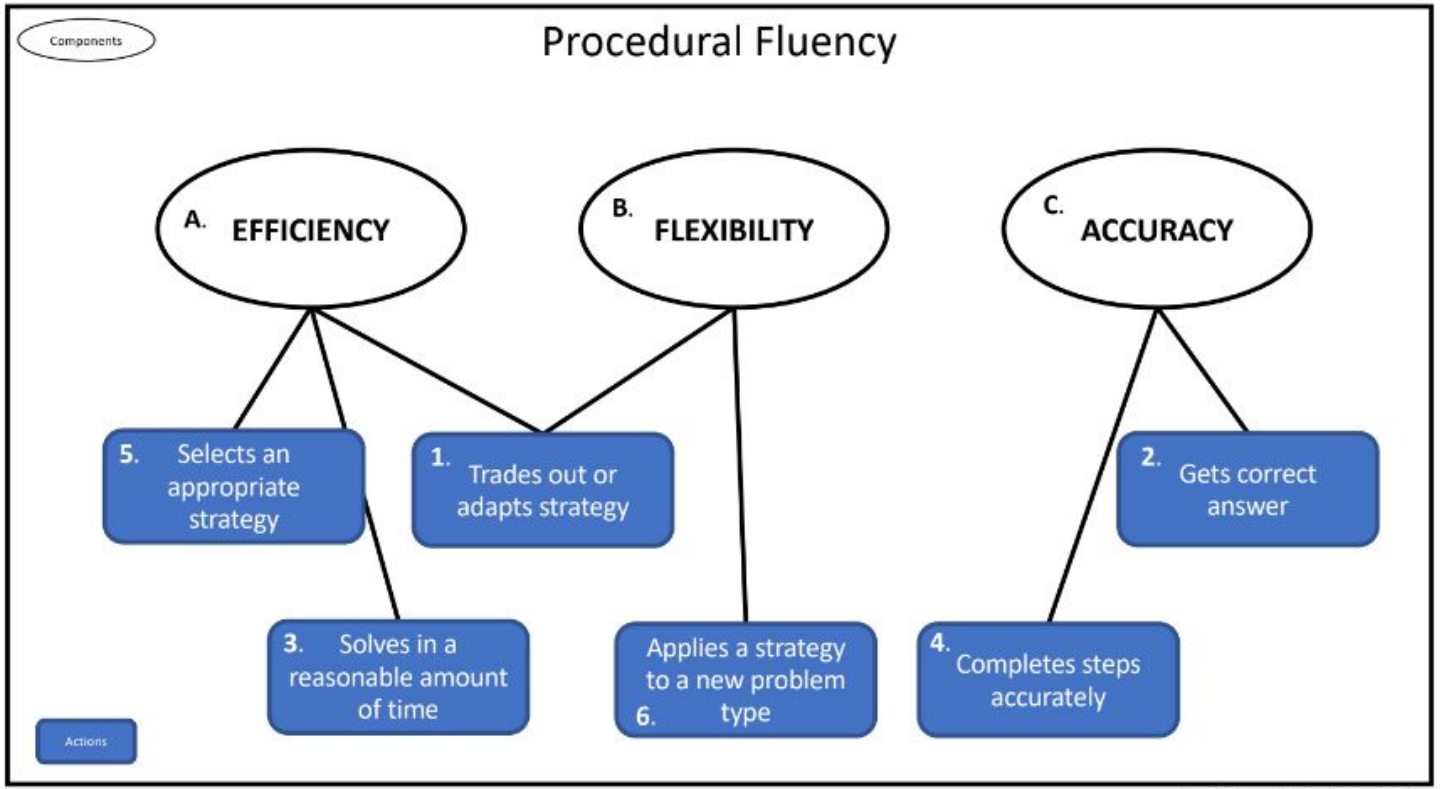
$$0.9 + 0.7 = 1 + 0.6 = 1.06$$

$$\frac{9}{10} + \frac{7}{10} = 1 + \frac{6}{10} = 1\frac{3}{5}$$

Reasoning Strategies

STRATEGY	RELEVANT OPERATIONS
1. Count On/Count Back	Addition & Subtraction
2. Make Tens	Addition
3. Use Partials	Addition, Subtraction, Multiplication, & Division
4. Break Apart to Multiply	Multiplication
5. Halve & Double	Multiplication
6. Compensation	Addition, Subtraction & Multiplication
7. Use an Inverse Relationship	Subtraction & Division

Procedural Fluency



How would you solve this?

1,435 ÷ 7

```
   205  R0
7 ) 1435
   -14
    03
    -0
     35
    -35
      0
```

1,000 ÷ 7
400 ÷ 7
30 ÷ 7
5 ÷ 7
..... ??

1435	
−700	100
735	
−700	100
35	
−35	5
0	205

1,400 ÷ 7 = 200
35 ÷ 7 = 5
205

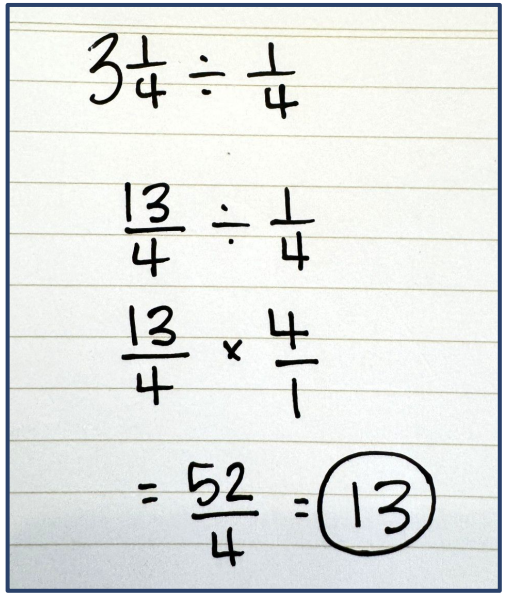
PROCEDURAL FLUENCY: 4 DECLARATIONS

1. Basic facts should be taught using number relationships and reasoning strategies, not memorization.
2. Procedural fluency requires having a repertoire of strategies.
3. **Conceptual understanding** must **precede** and **coincide** with instruction on procedures.

How would you solve these?

Find the quotient

$3\frac{1}{4} \div \frac{1}{4}$

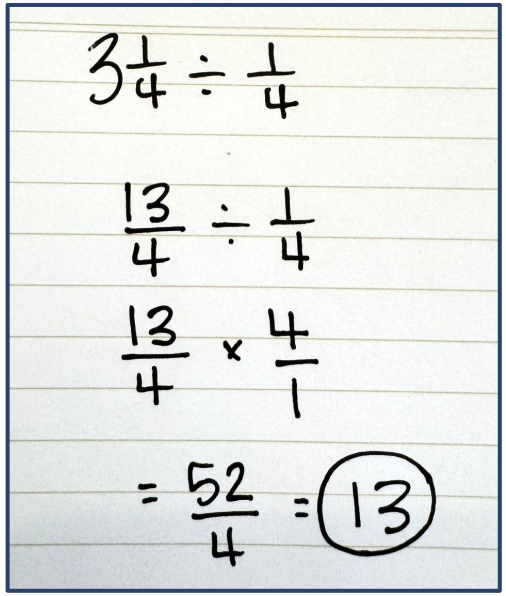


How many ¼ fit into 3¼?

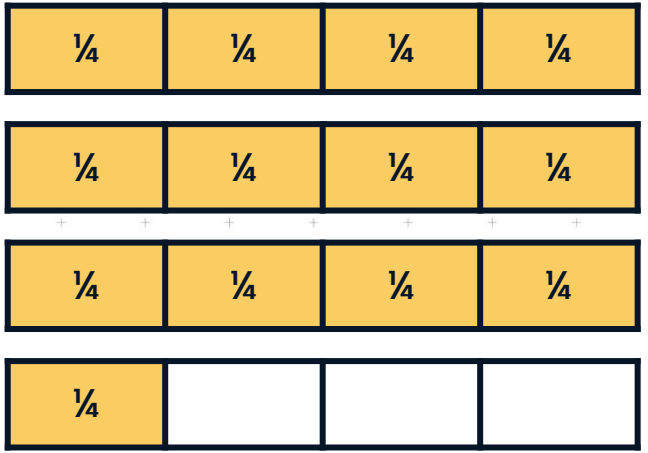
How would you solve these?

Find the quotient

$3\frac{1}{4} \div \frac{1}{4}$



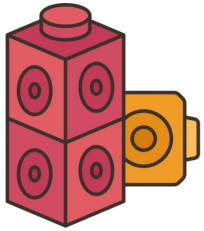
How many ¼ fit into 3¼?



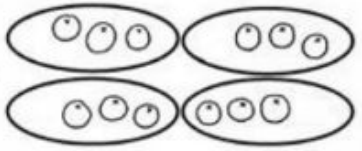
"Much research suggests that the instructional sequence of moving from physical representations *through* visual representation *to* symbolic representations leads to **significant gains** in math learning and understanding, particularly for students struggling in mathematics."

Visible Learning for Mathematics

CONCRETE



PICTORIAL



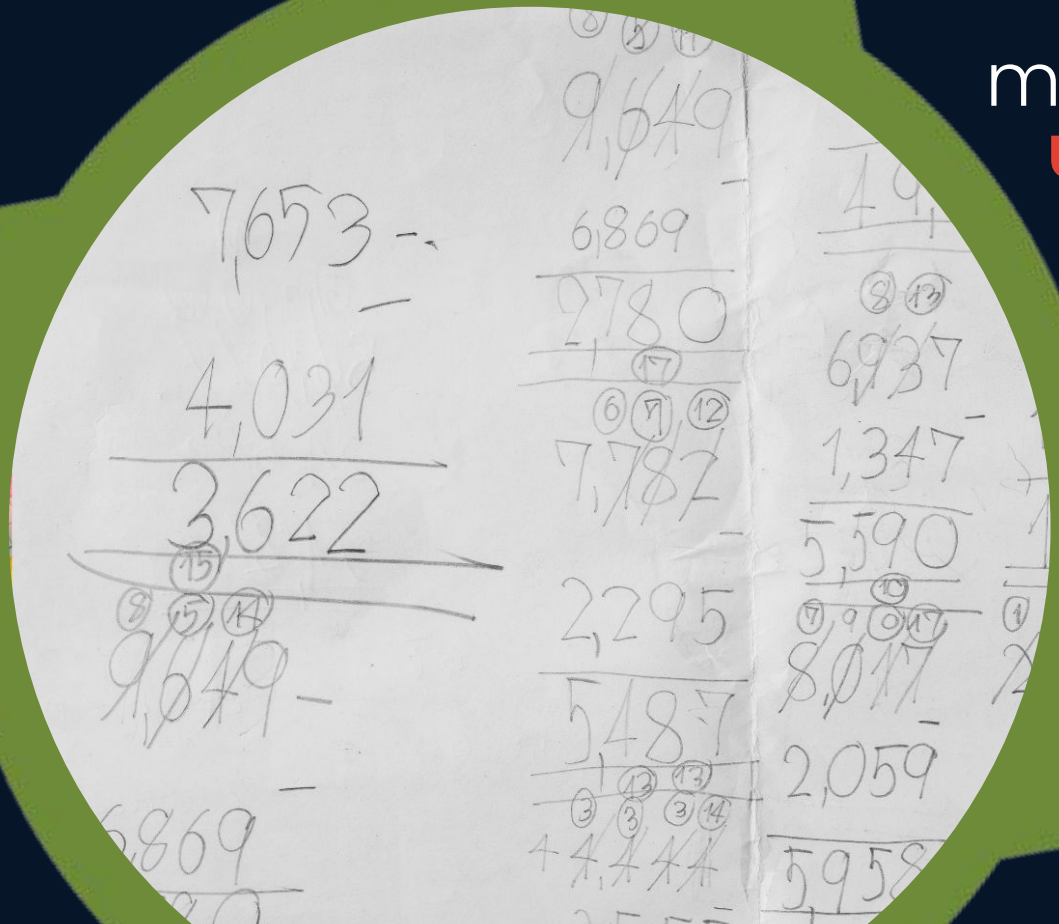
ABSTRACT

$12 \div 4 = 3$

What happens when we don't follow this order?

Conceptual foundations and reasoning strategies deepen conceptual understanding; **memorizing an algorithm does not.**

When students use a procedure they do not understand, they are more likely to make errors and fail to notice when the answer does not make sense.

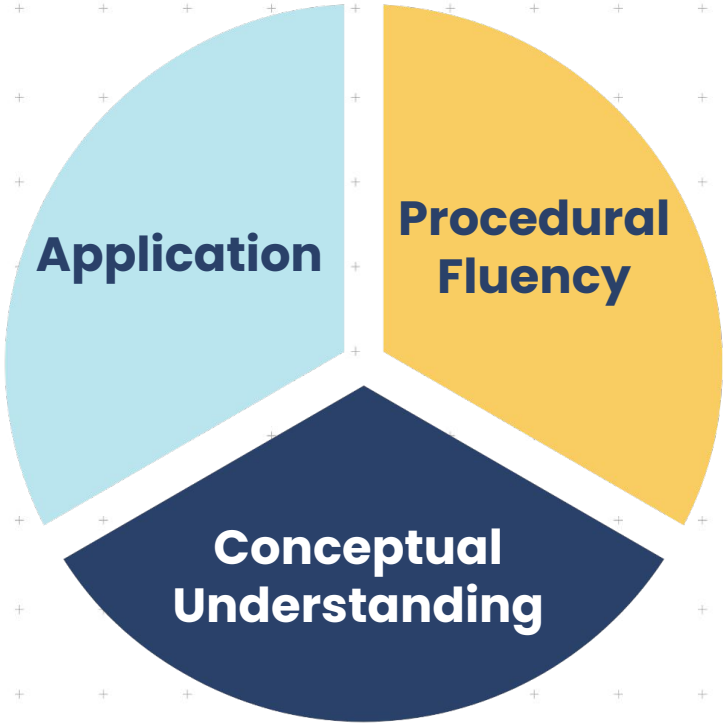


"A rush to the standard algorithm and memorizing procedures **undermines** students' confidence and may cause math anxiety, which negatively impacts student achievement."

-Boaler, Jameson, Ramirez

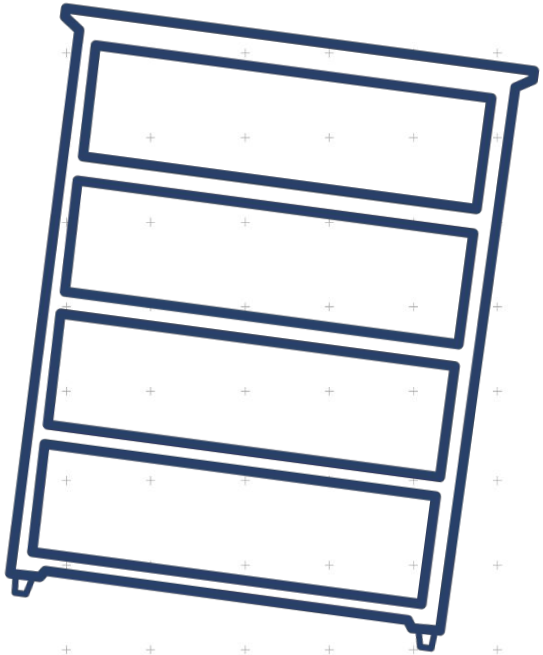
Conceptual Understanding

“Procedures grounded in understanding are better retained and applied.”
(Fuson 2005)



Do we accidentally create “mimickers” instead of mathematicians?

If I can build a bookshelf by following explicit steps and directions, am I fluent at building furniture?



Notebook or Neighbor

Has your definition of fluency changed? How so?

COMPASS

WHAT'S NOT WORKING?

“My students don’t read the directions.”

“They had it last week and now they don’t.”

“The algorithm makes the most sense to my students.”

“They always need me to get them started.”

“They can do it with manupulatives, but don’t understand when I take them away.”

PURCHASED RESOURCE

ASSESSMENTS

Too Long!

Too guided/scaffolded

Not organized by skill/outcome

Hard to analyze

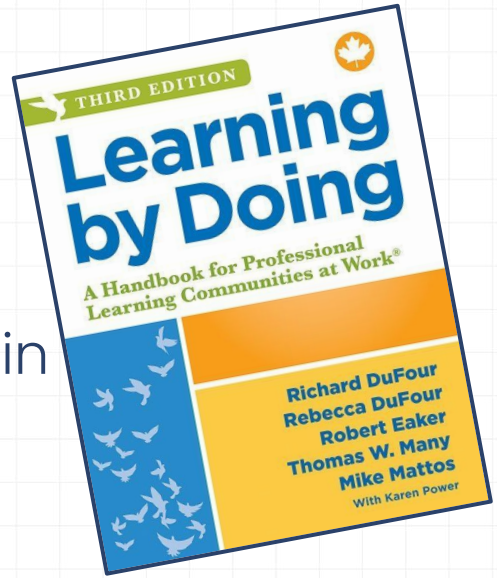
Multiple Choice

PURCHASED RESOURCE

PACING

FOCUS ON **PROFICIENCY** IN SKILLS-**NOT COVERAGE!**

Strict/rigid pacing guides promote "covering" the curriculum as opposed to **teaching the curriculum.**



"Constructive pacing guides assume differences in teachers, students, and school contexts. They adjust expectations through frequent revisions based on *input from teachers(CFA's)."*

Jane David (2008)

PURCHASED RESOURCE

PRACTICE

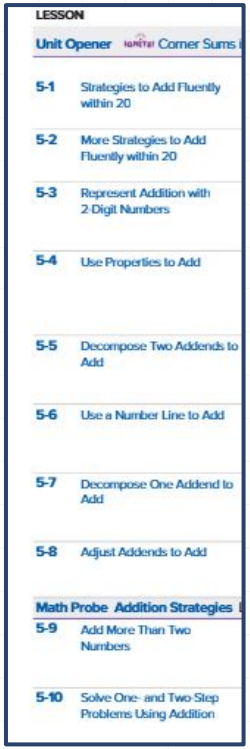
-Too much practice on computers!

-Often provides practice with specific strategies & focused solely on accuracy

-Practice often stops when the unit ends… **Students need more than "15 days" to become fluent!**

-Practice provided is typically designed to be independent and in a workbook… **Practice can be fun!**

We need to provide practice analyzing problems, predicting reasonable answers, and choosing an appropriate strategy.



Reveal Math 2022.

Cross multiply

Move the decimal

Butterfly Method

Just add a zero

Carry the one

Cancel

Flip and multiply

Rainbow Method

“More” means addition

MATH TRICKS

THEY DON'T STICK!

Conceptual Understanding → Procedural Fluency

$$\frac{5}{8} \div \frac{2}{3} \qquad \frac{5}{x} = \frac{2}{3} \qquad \frac{3}{5} \times \frac{2}{3}$$

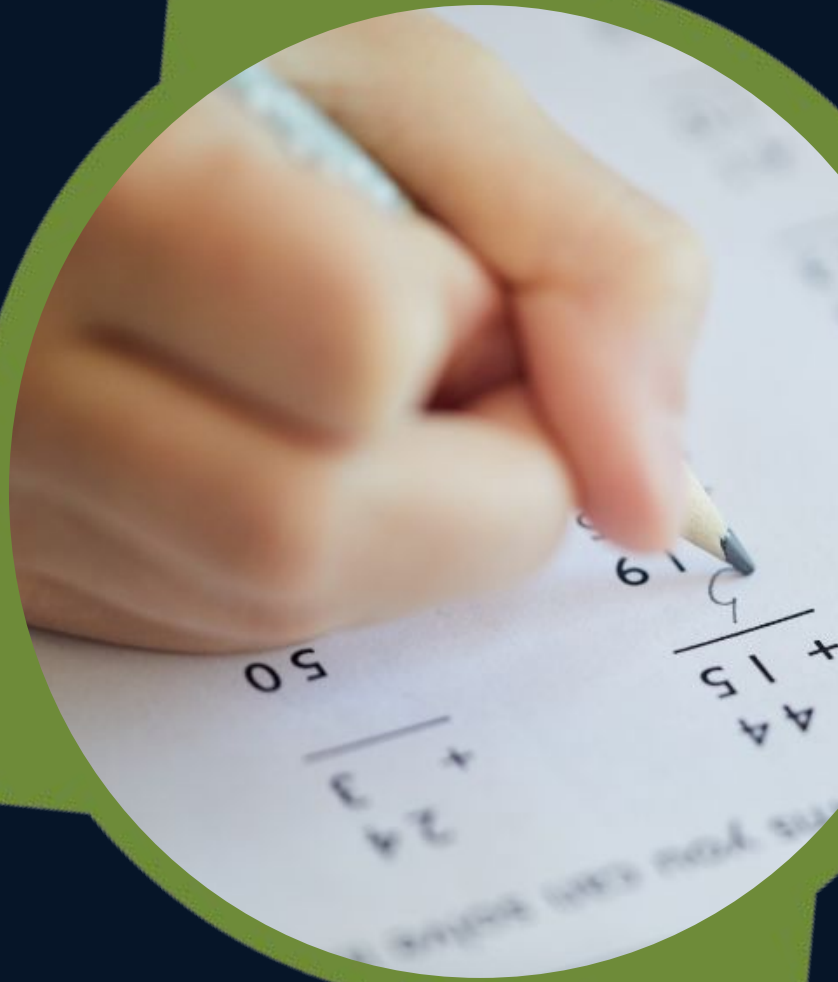
"How did my teacher do that?"

"Which of my strategies are a good fit for this problem?"

NCTM - Procedural Fluency

“It is possible that the reason many people think that fluency is the same as mastery or automaticity is because **accuracy** is the only thing we assess.”

-Jennifer M. Bay-Williams
& John J. SanGiovanni



COMMON MISCONCEPTION

FLUENCY = AUTOMATICITY

TIMED TESTS DO NOT ASSESS FLUENCY!

Timed tests only assess:

- Efficiency(memorization?)
- Accuracy

Automaticity is just one component...true fluency also includes decision making(conceptual understanding)!

COMMON MISCONCEPTION

FLUENCY = AUTOMATICITY

Could timed tests be doing more harm than good?

"Timed math tests can discourage students, leading to a math anxiety and a long-term fear of the subject."

–Jo Boaler, 2014

COMMON MISCONCEPTION

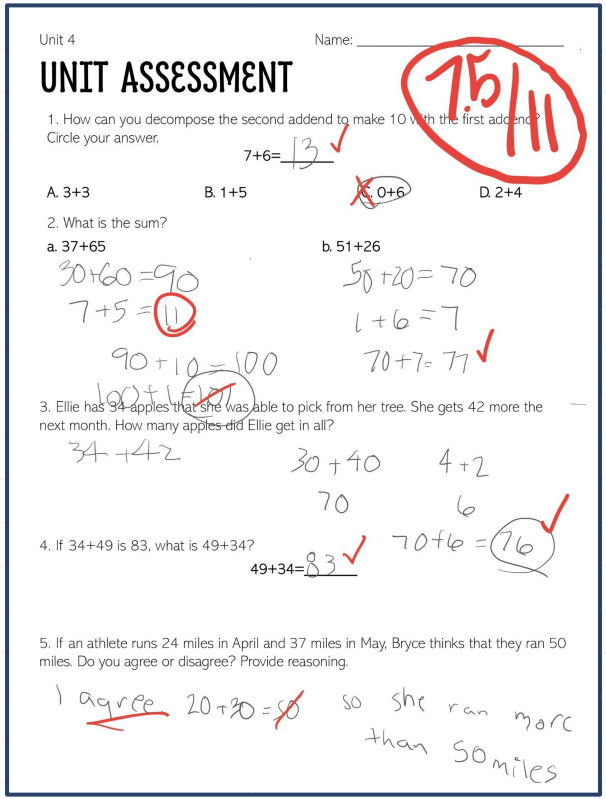
FLUENCY = AUTOMATICITY

How could we use timed tests differently?

-Don’t share time limit with students
-**Use it as a diagnostic to further investigate**
-Use for student self-assessment
-Two pen test
-Don’t expect completion

COMMON MISCONCEPTION

CORRECT ANSWERS MATTER MOST



Unit 4 Name: _____

UNIT ASSESSMENT

7.5/11

1. How can you decompose the second addend to make 10 with the first addend? Circle your answer.

7+6= 13 ✓

A. 3+3 B. 1+5 C. 0+6 D. 2+4

2. What is the sum?

a. 37+65

30+60=90

7+5=11

90+10=100

100+11=111

b. 51+26

50+20=70

1+6=7

70+7=77 ✓

3. Ellie has 34 apples that she was able to pick from her tree. She gets 42 more the next month. How many apples did Ellie get in all?

34+42

30+40 4+2

70 6

70+6=76 ✓

4. If 34+49 is 83, what is 49+34?

49+34=83 ✓

5. If an athlete runs 24 miles in April and 37 miles in May, Bryce thinks that they ran 50 miles. Do you agree or disagree? Provide reasoning.

I agree 20+30=50 so she ran more than 50 miles

Success Criteria:

- ✓ I can add single digit numbers.
- ✓ I can decompose numbers into tens and ones.
- ✓ I can add two numbers within 100.
- ❑ I can find sums accurately.
- ✓ I know that more tells me to add.
- ❑ I can solve story problems.

Grouping students by accuracy alone will prevent us from targeting specific skills that are missing!

How would you solve this?

535+398

```
  1 1
  535
 +398
 ----
  933
```

500+300 + 90+30 + 8+5

800 + 120 + 13

933

533+400

933

535+400=935

935-2=933

COMMON STRUGGLE

"NEW MATH" IS CONFUSING

The standard algorithm is only one strategy.

There are many instances where the standard algorithm is not the most **efficient**.

This might be teacher's comfort zone!

COMMON STRUGGLE

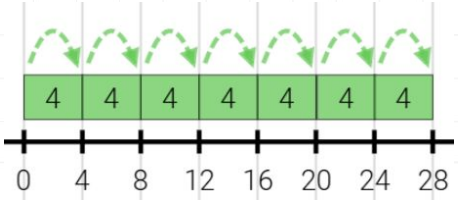
"NEW MATH" IS CONFUSING

Many elementary teachers enter the profession with limited preparation in math (NCTQ, 2022), and gaps in content knowledge often lead to instructional uncertainty and anxiety.

COMMON STRUGGLE

TOO MANY STRATEGIES!

4 x 7=_



Visual representations are not strategies! They are meant to deepen conceptual understanding.

Doubles

4x7=2x7+2x7=

14+14=28

Break Apart

4x7=4x5+4x2=

20+8=28

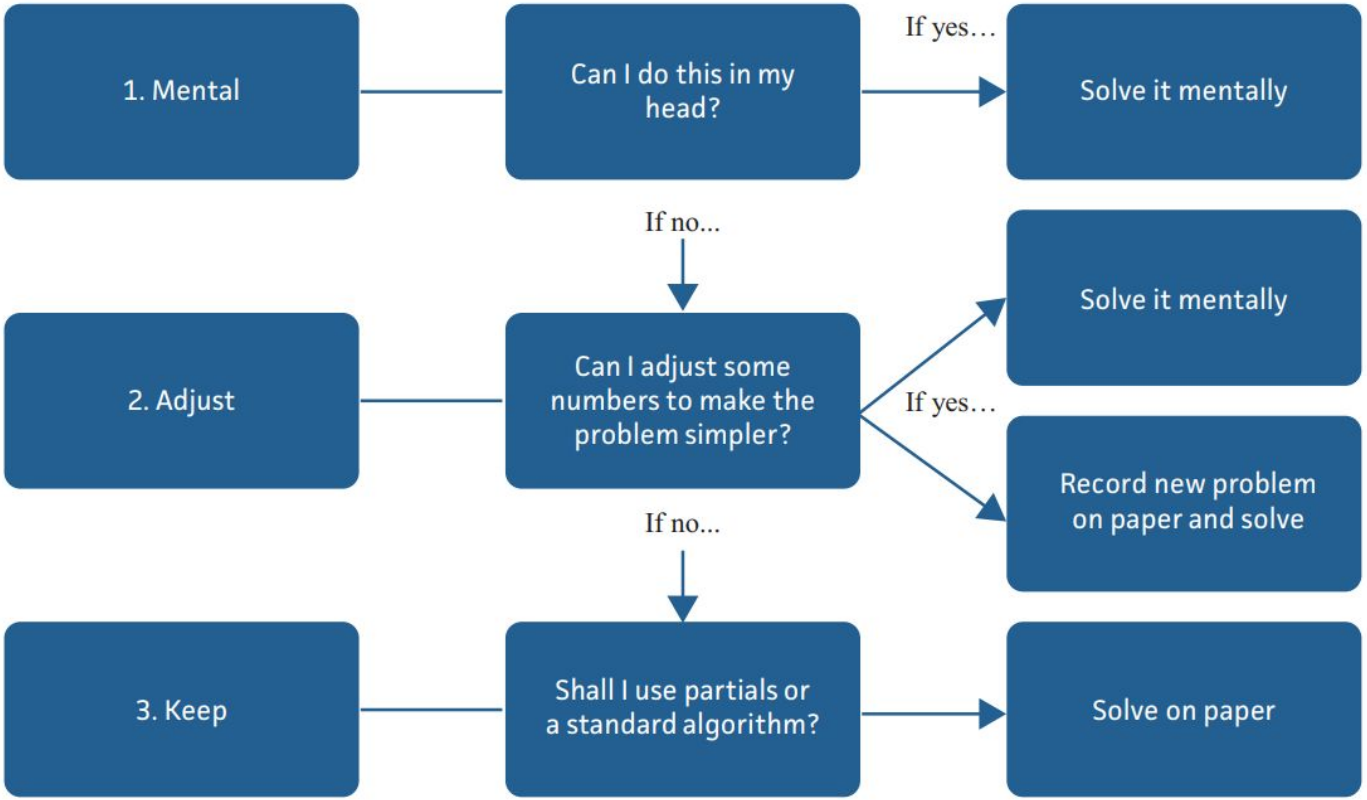
Add/Subtract a Group

4x7=4x6+4= 24+4=28

Part of being fluent is having **flexibility** within strategies.

Figuring Out Fluency. Williams & SanGiovanni

METACOGNITIVE PROCESS FOR SELECTING A STRATEGY

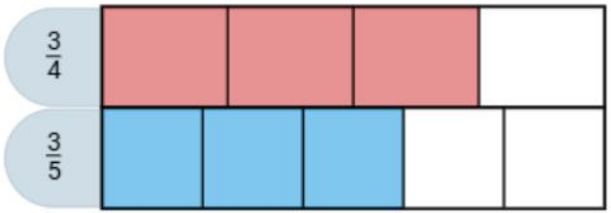


COMMON STRUGGLE

MANIPULATIVES AREN'T WORKING

"They can do it with the manipulatives, but then can't when I take them away."

Manipulatives are meant to deepen conceptual understanding. They are not meant to simply solve problems.



$$\frac{3}{4} \bigcirc \frac{3}{5}$$

COMMON STRUGGLE

CENTERS(SMALL GROUP) ARE TOO MUCH WORK!

Too much to plan!

Too many transitions!

Hard to manage students!

Too many materials to prepare!

COMMON STRUGGLE

CENTERS(SMALL GROUP) ARE TOO MUCH WORK!

- Centers can span the course of many days
  - Centers should be repeated
- Avoid physical movement until you're ready
- Teacher does not have to be a center
  - **Work up to this!**
- Resources should be shared across multiple grade levels.
- Centers can be designed so the materials stay the same and the task changes
- Utilize some "low maintenance" centers.

Notebook or Neighbor

Which of these common struggles do you experience in your school?

COMPASS

WHAT ACTUALLY BUILDS FLUENCY

“My students don’t know…”

…basic addition
…fractions
…their multiplication facts
…the standard algorithm

What DO they know? SHOULD they know it?

Do these students need the same intervention?

STUDENT A

Struggling with their multiplication facts, including their 2's.

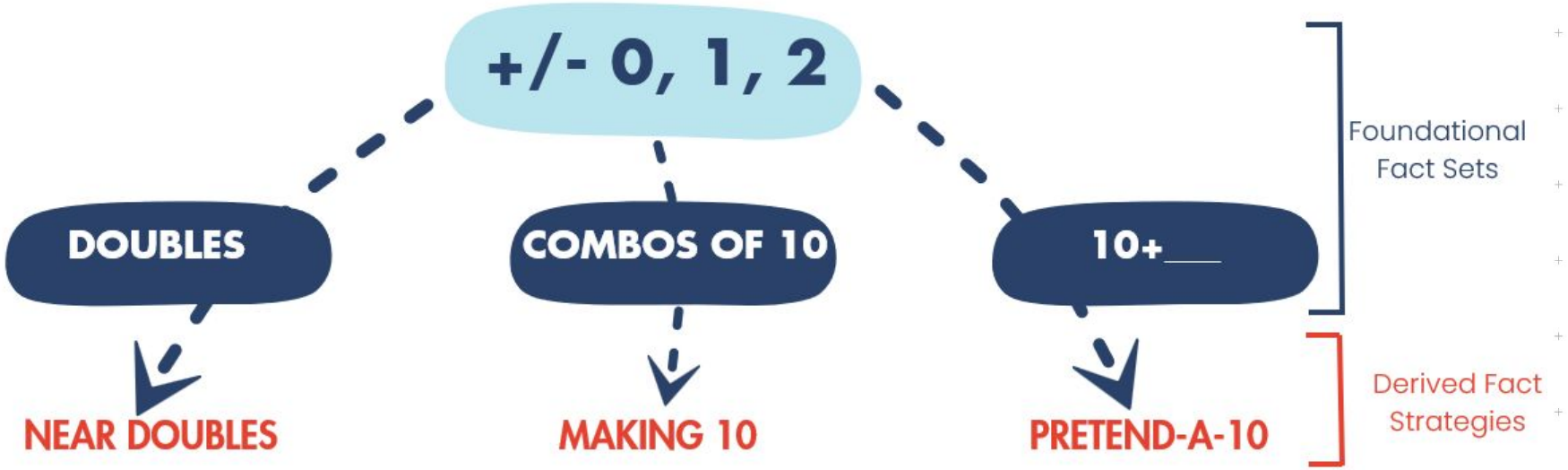
STUDENT B

Successful with most multiplication facts, but struggling on 7's.

STUDENT C

Unable to finish the assessment in time, but all correct.

Addition Learning Progression



2.OA.B.2.a

I AM LEARNING TO...

fluently add within 20.

1

I KNOW HOW TO ADD ZERO, ONE, & TWO.

0+0=? 1+1=? 1+2=?

0 1 2

2

I KNOW THE PARTS OF AN ADDITION EQUATION.

12 + 4 = 16

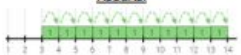
ADDEND ADDEND SUM

3

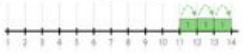
I CAN COUNT ON TO FIND A SUM.

3 + 11 = 14

I CAN COUNT ON FROM THE FIRST ADDEND.

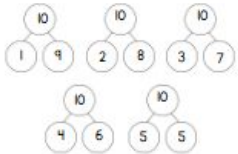


I CAN COUNT ON FROM THE LARGEST ADDEND.



5

I KNOW MY FACT FAMILIES OF 10.



6

I CAN ADD WITHIN 10.

7

I KNOW MY DOUBLES UP TO 10.

1+1=	6+6
2+2=	7+7=
3+3=	8+8=
4+4=	9+9=
5+5=	10+10=

8

I CAN ADD WITHIN 20 EFFICIENTLY.

COUNTING ON

DOUBLES

6+6=12

9+9=18

MAKE A TEN

7 + 5 = 12

3 2

NEAR DOUBLES

6 + 7 = 13

6 1

2.NBT.5.8

I AM LEARNING TO...

add within 100.

1

I CAN ADD WITHIN 20 EFFICIENTLY.

COUNTING ON

DOUBLES

6+6=12

9+9=18

MAKE A TEN

7 + 5 = 12

3 2

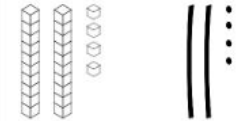
NEAR DOUBLES

6 + 7 = 13

6 1

2

I CAN REPRESENT ADDITION EQUATIONS USING BASE TEN BLOCKS.



3

I UNDERSTAND THE COMMUTATIVE PROPERTY AND CAN APPLY IT.

23 + 42 = 65

42 + 23 = 65

4

I CAN DECOMPOSE TO FIND A SUM. (PARTIAL SUMS)

51 + 28

50 + 1 20 + 8

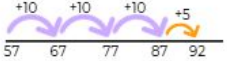
50 + 20 = 70 1 + 8 = 9

70 + 9 = 79

5

I CAN COUNT ON TO FIND A SUM. (NUMBER LINE)

57+35=92



6

I CAN ADJUST ADDENDS. (COMPENSATION)

27 + 38 = 65

+3 −3

30 + 35 = 65

7

I CAN SOLVE PROBLEMS WITH REGROUPING.

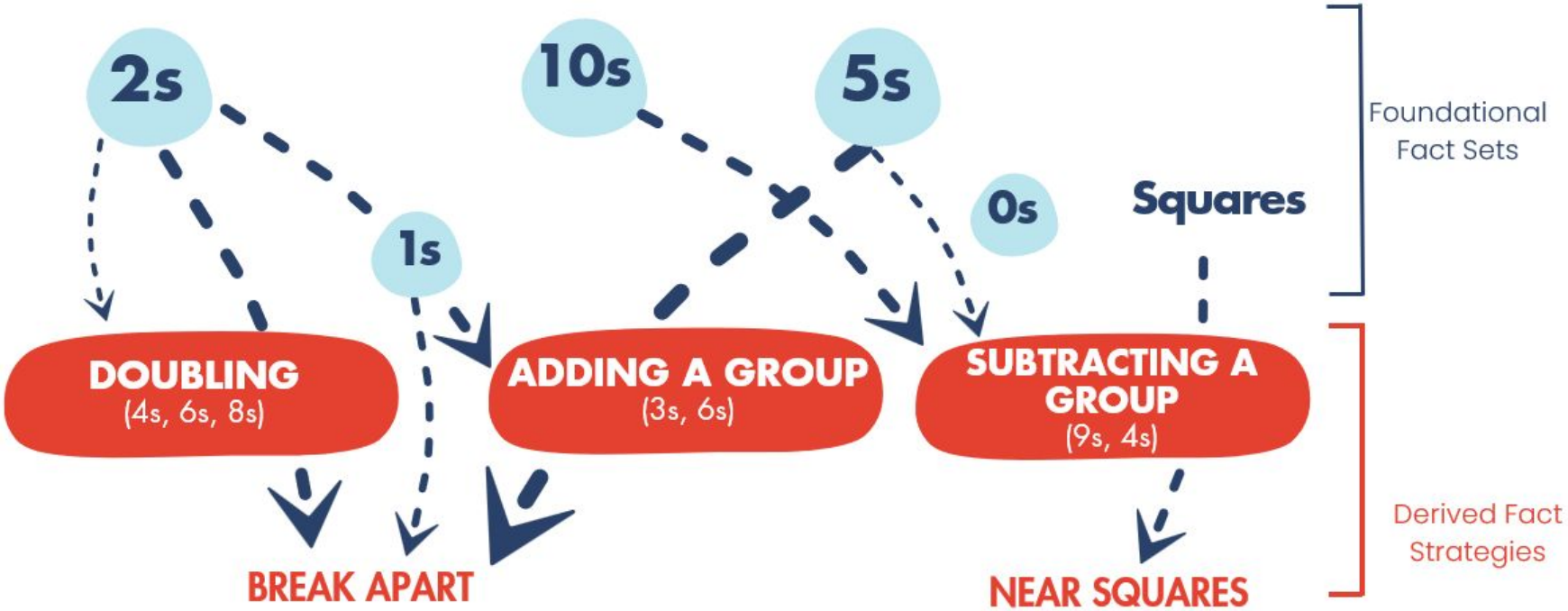
8

I CAN ADD WITHIN 100 USING DIFFERENT STRATEGIES.

DECOMPOSING ADJUST ADDENDS

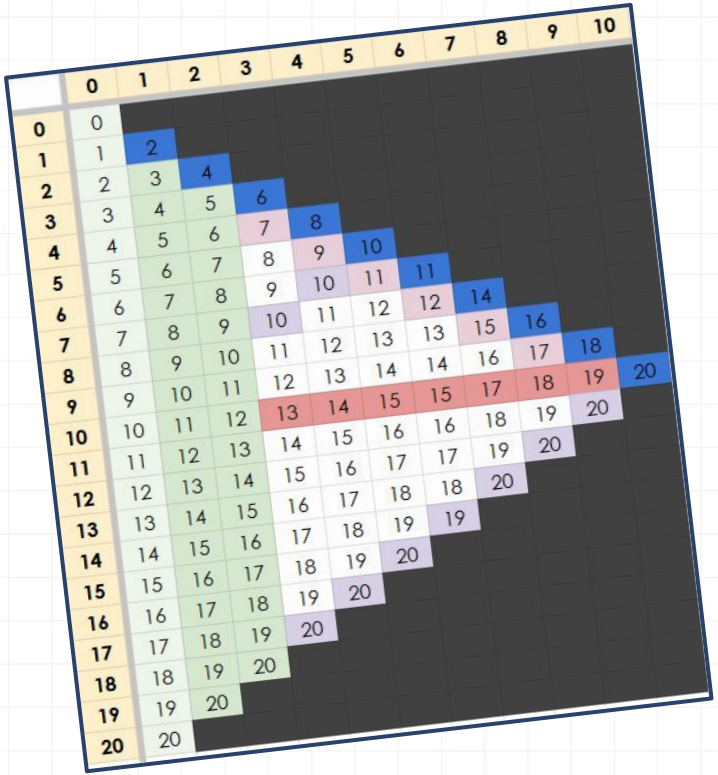
COUNTING ON

Multiplication Learning Progression

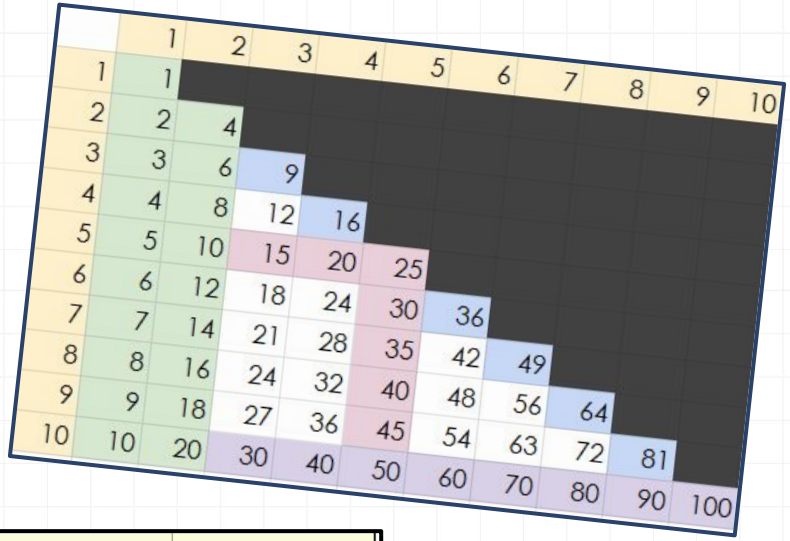


What facts DO they know?

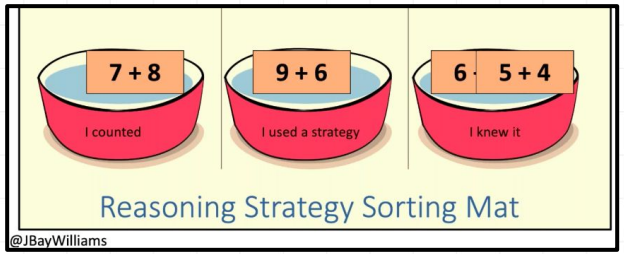
Intervene differently depending on the facts they don't!



	0	1	2	3	4	5	6	7	8	9	10
0	0										
1	1	2									
2	2	3	4								
3	3	4	5	6							
4	4	5	6	7	8						
5	5	6	7	8	9	10					
6	6	7	8	9	10	11	12				
7	7	8	9	10	11	12	13	14			
8	8	9	10	11	12	13	14	15	16		
9	9	10	11	12	13	14	15	16	17	18	
10	10	11	12	13	14	15	15	17	18	19	20
11	11	12	13	14	15	16	16	18	19	20	
12	12	13	14	15	16	17	17	19	20		
13	13	14	15	16	17	18	18	20			
14	14	15	16	17	18	19	19				
15	15	16	17	18	19	20					
16	16	17	18	19	20						
17	17	18	19	20							
18	18	19	20								
19	19	20									
20	20										



	1	2	3	4	5	6	7	8	9	10
1	1									
2	2	4								
3	3	6	9							
4	4	8	12	16						
5	5	10	15	20	25					
6	6	12	18	24	30	36				
7	7	14	21	28	35	42	49			
8	8	16	24	32	40	48	56	64		
9	9	18	27	36	45	54	63	72	81	
10	10	20	30	40	50	60	70	80	90	100



South Dakota K-5 Math Fluency Progression

Foundations	Conceptual understanding	**Fluently**	Skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
Developing	Conceptual understanding with strategies to operate	**Memorization**	Quick, effortless recall of facts.

Strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Four operations with **whole numbers.**

Kindergarten

Addition

- Within 20: Fluentlly add and subtract **within 5.**
- Within 20: Add and subtract **within 10** using objects and drawings.

Subtraction

- Within 20: Fluentlly add and subtract **within 5.**
- Within 20: Add and subtract **within 10** using objects and drawings.

1st Grade

Addition

- Within 20: Fluentlly add and subtract **within 10.**
- Within 20: Add and subtract **within 20.**
- Within 100: Add **within 100.**

Subtraction

- Within 20: Fluentlly add and subtract **within 10.**
- Within 20: Add and subtract **within 20.**
- Within 100: Subtract **within 100.** (multiples of 10)

2nd Grade

Addition

- Within 20: Fluently add/subtract using mental strategies **within 20.**
- Within 20: By end of year know all **sums within 20** from **memory.**
- Within 100: Fluently add/subtract **within 100.**
- Within 1000: Add/subtract **within 1000.**

Subtraction

- Within 20: Fluently add/subtract using mental strategies **within 20.**
- Within 20: By end of year know all **sums within 20** from **memory.**
- Within 100: Fluently add/subtract **within 100.**
- Within 1000: Add/subtract **within 1000.**

Multiplication

- Within 20: Foundations only: equal groups (objects to 20) and arrays (up to 5x5)

3rd Grade

Addition

- Within 1000: Fluently add/subtract **within 1000. (Algorithm included)**
- 1000+: Fluently add/subtract **within 1000. (Algorithm included)**

Subtraction

- Within 1000: Fluently add/subtract **within 1000. (Algorithm included)**
- 1000+: Fluently add/subtract **within 1000. (Algorithm included)**

Multiplication

- Within 100: Fluently multiply/divide **within 100.**
- Within 1000: Multiply **4x1-digit** and **2x2-digit.**

Division

- Within 100: Fluently multiply/divide **within 100.**
- Within 1000: Divide **4 by 1-digit.** (Whole digit remainders)

4th Grade

Addition

- Within 1000: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**
- 1000+: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**

Subtraction

- Within 1000: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**
- 1000+: Fluently add/subtract **multi-digit whole numbers. (Algorithm included)**

Multiplication

- Within 100: Know from **memory** all products of **2 one digit numbers.**
- Within 1000: Multiply **4x1-digit** and **2x2-digit.**

Division

- Within 100: Divide **4 by 1-digit.** (Whole digit remainders)
- Within 1000: Divide **4 by 1-digit.** (Whole digit remainders)

5th Grade

Addition

- Within 1000: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**
- 1000+: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**

Subtraction

- Within 1000: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**
- 1000+: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**

Multiplication

- Within 1000: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**
- 1000+: Fluently multiply **multi-digit** whole numbers. **(Algorithm included)**

Division

- Within 1000: Divide **4 by 2-digit numbers.**
- 1000+: Divide **4 by 2-digit numbers.**

6th Grade

Addition

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Subtraction

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Multiplication

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Division

- 1000+: Fluently divide **multi-digit** whole numbers. **(Algorithm included)**

Four operations with **decimals.**

Kindergarten

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

1st Grade

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

2nd Grade

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

Multiplication

- Decimals: Exposure to decimals in terms of money

3rd Grade

Addition

- Decimals: Exposure to decimals in terms of money

Subtraction

- Decimals: Exposure to decimals in terms of money

Multiplication

- Decimals: Exposure to decimals in terms of money

Division

- Decimals: Exposure to decimals in terms of money

4th Grade

Addition

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

Subtraction

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

Multiplication

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

Division

- Decimals: Read, write, and compare fractions as decimals. (denominators of 10 and 100)

5th Grade

Addition

- Decimals: Add/Subtract decimals to the hundredth.

Subtraction

- Decimals: Add/Subtract decimals to the hundredth.

Multiplication

- Decimals: Multiply/Divide decimals to the hundredth.

Division

- Decimals: Multiply/Divide decimals to the hundredth.

6th Grade

Addition

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Subtraction

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Multiplication

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Division

- Decimals: Fluently add, subtract, multiply, divide **multi-digit decimals. (Algorithm included)**

Four operations with **fractions.**

Kindergarten

Addition

- Represent(visually & verbally) **halves & fourths.**

Subtraction

- Represent(visually & verbally) **halves & fourths.**

1st Grade

Addition

- Represent(visually & verbally) **halves, thirds, & fourths.** Whole Numbers

Subtraction

- Represent(visually & verbally) **halves, thirds, & fourths.** Whole Numbers

2nd Grade

Addition

- Represent(visually, verbally, numerically) **halves, thirds, & fourths, sixths, eighths.** Whole Numbers, comparisons, equivalent, benchmarks

Subtraction

- Represent(visually, verbally, numerically) **halves, thirds, & fourths, sixths, eighths.** Whole Numbers, comparisons, equivalent, benchmarks

Multiplication

- Represent(visually, verbally, numerically) **halves, thirds, & fourths, sixths, eighths.** Whole Numbers, comparisons, equivalent, benchmarks

3rd Grade

Addition

- Add/Subtract fractions with **like denominators** (including mixed numbers).
- Add fractions with denominators 10 and 100.

Subtraction

- Add/Subtract fractions with **like denominators** (including mixed numbers).
- Add fractions with denominators 10 and 100.

Multiplication

- Multiply whole number by fraction.

4th Grade

Addition

- Add/Subtract fractions with **unlike denominators** (including mixed numbers).

Subtraction

- Add/Subtract fractions with **unlike denominators** (including mixed numbers).

Multiplication

- Multiply (whole number x fraction) & (fraction x fraction) (including mixed numbers)

Division

- Divide (whole number by unit fraction) & (unit fraction by whole number)

5th Grade

Addition

- Multiply/Divide fractions by fractions. (including mixed numbers)

Subtraction

- Multiply/Divide fractions by fractions. (including mixed numbers)

Multiplication

- Multiply/Divide fractions by fractions. (including mixed numbers)

Division

- Multiply/Divide fractions by fractions. (including mixed numbers)

6th Grade

Addition

- Multiply/Divide fractions by fractions. (including mixed numbers)

Subtraction

- Multiply/Divide fractions by fractions. (including mixed numbers)

Multiplication

- Multiply/Divide fractions by fractions. (including mixed numbers)

Division

- Multiply/Divide fractions by fractions. (including mixed numbers)

PROCEDURAL FLUENCY: 4 DECLARATIONS

1. Basic facts should be taught using number relationships and reasoning strategies, not memorization.
2. Procedural fluency requires having a repertoire of strategies.
3. Conceptual understanding must precede and coincide with instruction on procedures.
4. **Assessing** must attend to fluency components and the learner. Assessments often assess accuracy, **neglecting efficiency and flexibility**.

How can we assess fluency?

PAPER ASSESSMENT

-Keep questions open ended.
-Describe if a worked example is efficient or not.
-Ask students to use learned strategies at least once.

INTERVIEW

-Ask students to solve and then share with you how.
-Smile & Encourage, but to not teach, correct, or confirm.
-For some, ask how they know their answer is reasonable

OBSERVATION

-Observe during small group tasks, games, or centers.
-Use an observation checklist
-All students don't have to be observed in one day!

What does proficiency in fluency look like?

BEGINNING 1	DEVELOPING 2	EMERGING 3	ACCOMPLISHED 4
Knows one algorithm or strategy but continues to get stuck or make errors.	Demonstrates efficiency and accuracy with at least one strategy/ algorithm, but does not stop to think if there is a more efficient possibility.	Demonstrates efficiency and accuracy with several strategies, and sometimes selects an efficient strategy, though still figuring out when to use and not use a strategy.	Demonstrates efficiency and accuracy with several strategies and is adept at matching problems with efficient strategies (knowing when to use each and when not to).

Figuring Out Fluency. Williams & SanGiovanni

Interview Assessment

Teacher 1 Name

Notes Key

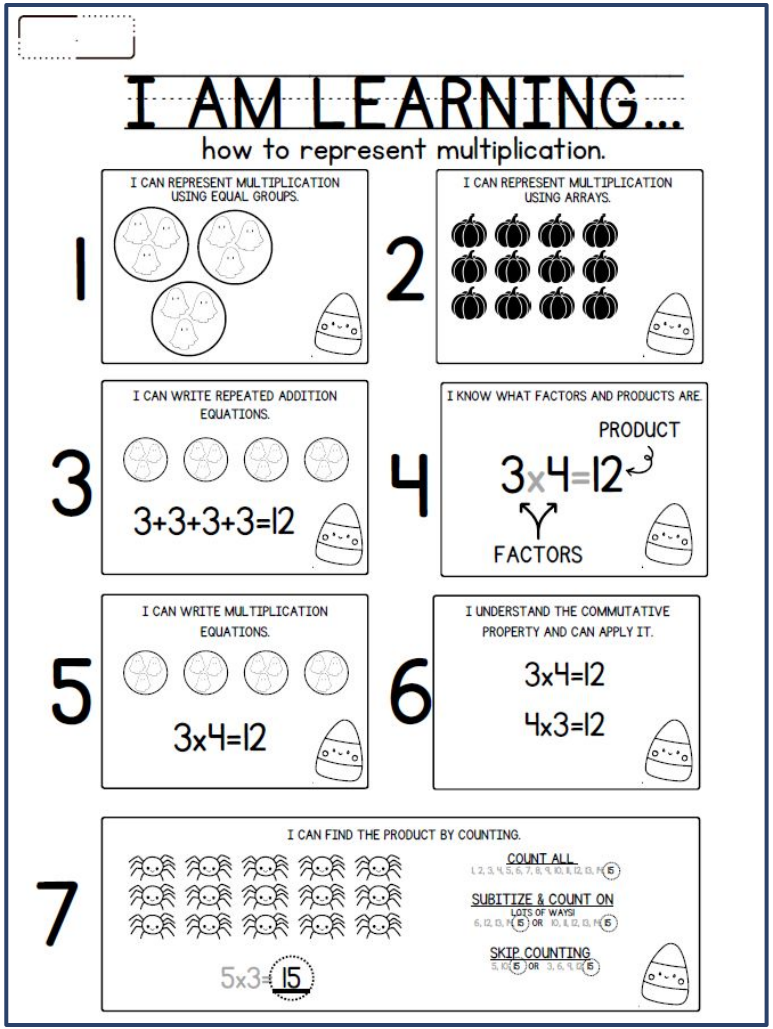
F-Fingers
CA-Count all
CO-Count On (Larger or 1st #)
D-Doubles (+/- 1 or 2)
MT-Make Ten

A - Automatic	After reading the equation, student produces answer effortlessly
S- Strategy	Efficiently produces the answer using a strategy (within 5 seconds)
P/C - Prolonged/ Counting	Counted all, count on from first addend, student needed additional thinking time, self corrects, etc.
X - Incorrect or No Response	

Addition within 20

	Foundational Facts			10+_ & 10's Facts				Doubles		Close Addends		Addends			TOTAL		Strategy Follow-up Questions		
	0+12=_	1+13=_	2+17=_	10+7=_	3+7=_	6+4=_	5+5=_	4+4=_	7+7=_	5+6=_	3+4=_	9+7=_	8+4=_	7+5=_	# Correct	# Automatic			
Student 1	A	A	A	A	A	A	A	A	A	A	A	A	A	A	14	14	☑	**10+_** (10+9? 10+2?)	
															4-Exceeding		☑	**10's Facts** (8+2? 1+9?)	
																	☑	**Doubles** (8+8? 9+9?)	
Student 2	A	A	P/C	A	P/C	P/C	A	A	A	S	A	P/C	P/C	A	14	8	☑	**10+_** (10+9? 10+2?)	Needs practice with make ten strategy and 10's facts
			CO		CO					D		CO	CO		2- Approaching		☐	**10's Facts** (8+2? 1+9?)	
																	☑	**Doubles** (8+8? 9+9?)	
Student 3	A	A	A	A	A	A	A	A	A	P/C	X	S	S	S	13	9	☑	**10+_** (10+9? 10+2?)	Needs more practice with near doubles strategy!
										CO		MT	MT	MT	2- Approaching		☑	**10's Facts** (8+2? 1+9?)	
																	☑	**Doubles** (8+8? 9+9?)	
Student 4	X	P/C	P/C	A	P/C	P/C	A	P/C	P/C	P/C	A	P/C	X	P/C	12	3	☑	**10+_** (10+9? 10+2?)	Primarily Counting On!
		CO	CO		CO	CO		CO	CO	CO		CO		CO	2- Approaching		☐	**10's Facts** (8+2? 1+9?)	
																	☐	**Doubles** (8+8? 9+9?)	

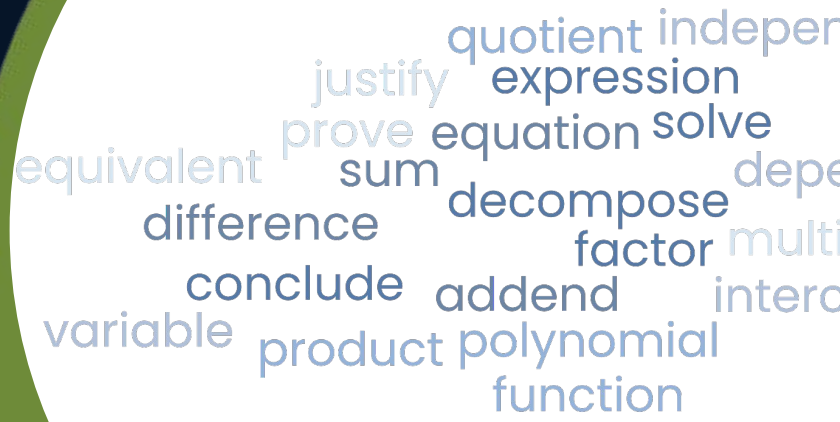
Conceptual Understanding



When teaching and assessing fluency, we must...

- Explicitly teach **vocabulary**
- Prioritize **student discourse**
- Model & practice **reasoning** strategies
- Provide low and **high level tasks**
- Demonstrate & practice **strategy choice**
- Assess true **fluency**

Math language is not its own lesson. It is in **every** lesson!

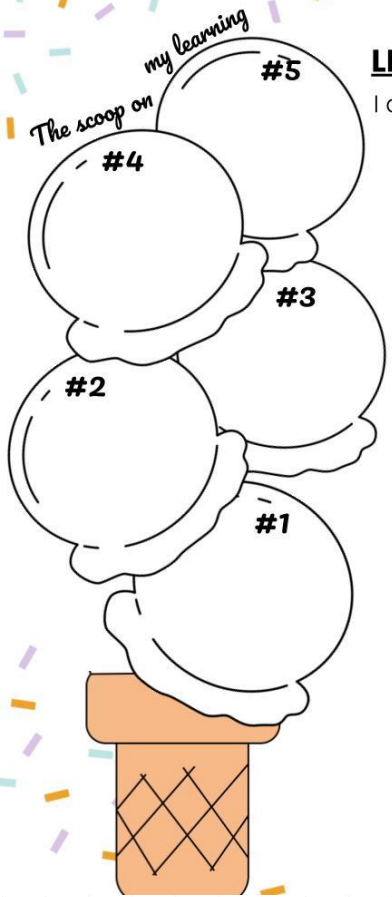


Vocabulary

Identify word parts

#3	I can decompose a composite figure into rectangles by drawing a partition.

3.MD.7



LEARNING INTENTION:

I am learning to find area of composite figures by decomposing into rectangles.

SUCCESS CRITERIA:

#1	I know what a composite figure is.
#2	I know what it means to decompose a figure with a partition and that they cannot overlap.
#3	I can decompose a composite figure into rectangles by drawing a partition.
#4	After I decompose the composite figure, I know to add the areas to find the total area.
#5	I can use multiplication and decomposing to find area in story problems, and include units in my answer.

Vocabulary

Identify word parts

#3	I can decompose a composite figure into rectangles by drawing a partition.

decompose

"undo"

partition

"part"

"The person doing the talking is the person doing the learning."

Cognitive Task Analysis

COGNITIVE TASK ANALYSIS

1.09

What it **is not:**

Following an algorithm.

Mimicking steps from a teacher or example.

Asking for immediate help when I get a wrong answer.

What it **is:**

Knowing when certain strategies are efficient.

Being able to identify and explain each step.

Using success criteria to identify exactly which steps are confusing.

How would you solve this?

Find the product.

5.5×12

Can I do this in my head?

5.5×12

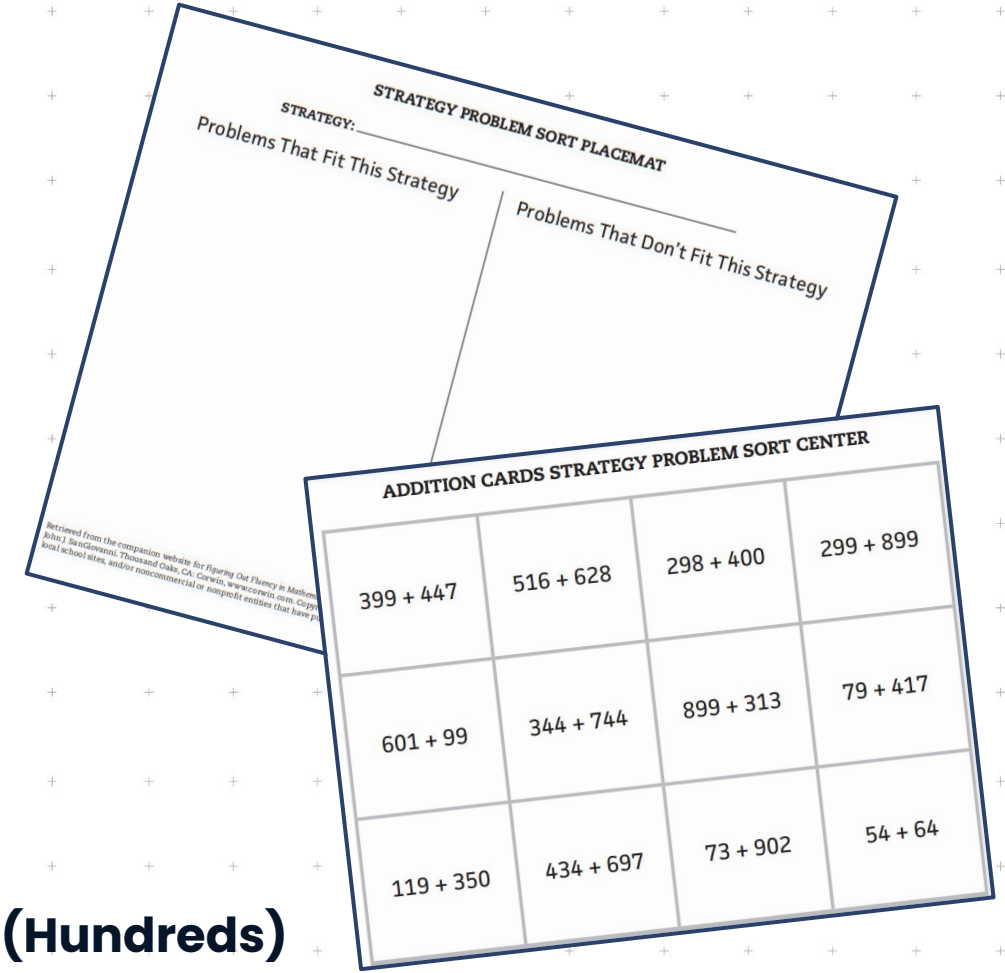
$= 5.5 \times (2 \times 6)$

$= 11 \times 6$

$= 66$

Focused

"Mixed practice happens after **focused** practice has taken place."



Make Tens(Hundreds)

Practice

We need students to practice...

Efficiently **Flexibly** **Accurately**

That One

(Which problems would solve with the standard algorithm?)

A. 0.25×48

B. 9×12.3

C. 3.7×4.1

D. 1.09×42.4

E. 8.5×0.2

F. 4.5×2

Reasoning Routines

You can do this every day! With little to no prep!

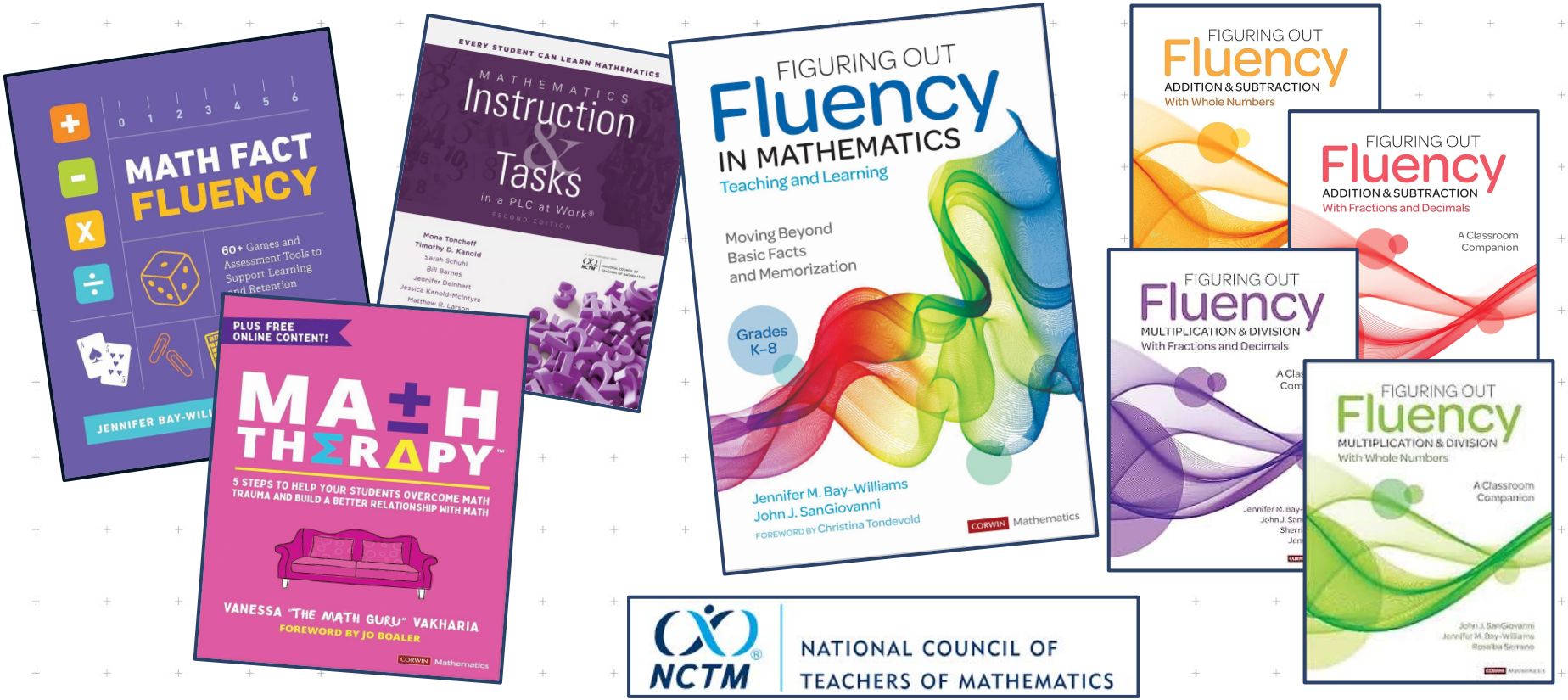
1. Estimate every problem!
2. Check reasonableness of answers.
3. Argumentation

   Susan read ⅘ of an hour and then ⅞ of an hour later that day. She claims that she read 2 hours. Do you agree or disagree?

It’s no secret that society doesn’t like math...

WE HAVE THE POWER TO CHANGE THAT.

RESOURCES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With Whole Numbers
FIGURING OUT Fluency
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